The Ideal Language Tradition

Frege and the Beginning of Philosophy

Logic & Philosophy before Frege

- Leibniz
- Kant
- Psychologism and the Basic Confusions
- The *Begriffsschrift*
- Frege's philosophy of language

Gottfried Wilhelm Leibniz (1646-1716)

- Idea of a "lingua characteristica", an artificial language based an alphabet of basic concepts.
- Since – as he thought – all basic statements would be of subject-predicate form ('S is P'), a conceptual analysis could reveal them as being true, in case P is found among the features S is composed of.

Gottfried Wilhelm Leibniz (1646-1716)

- Leibniz also held that this language should be "mathematical", i.e. represented in some symbolic form.
- With the help of the "calculus ractiocinator" it would then be possible to calculate the truths mechanically.

Immanuel Kant (1724-1804)

- The basic problem in philosophy of mathematics.
- Kant's solution
- Kant's solution in the lights of Aristotelian logic.
- Kant's solution in the lights of modern logic.

The basic problem in the philosophy of mathematics

- British empiricism and "continental" rationalism both did a poor job of accounting for mathematics.
- The empiricist gave a good explanation why mathematics seems to relate so perfectly to the world (in – for example – Euclidean geometry), but couldn't explain its necessity.
- The rationalists – on the other hand – could explain its necessity, but did a poor job explaining why mathematics is such a good tool to describe the empirical world

Kant's "solution"

- Kant tried to find a synthesis of the rationalists' and the empiricists' attempts to explain the nature of mathematics.
- How is mathematics knowable a priori and yet universally applicable – to all experiences – with incorrigible certainty?

Kant's notion of analyticity

- A universal proposition ('All S are P') is *analytic* if the predicate concept P is contained in the subject concept S; otherwise the proposition is *synthetic*.
- 'All bachelors are unmarried' is analytic if the concept of being unmarried is contained in the concept of bachelor.

Kant's notion of analyticity

- Conceptual analysis uncovers what is already implicit in concepts:
  - Analytic judgments could ... be called *elucidatory*. For they do not through the predicate add anything to the concept of the subject; rather they only dissect the concept, breaking it up into its component concepts which already been thought in it. (*Critique of Pure Reason*, B11)

Kant's notion of analyticity

- Thus, conceptual analysis does not yield new knowledge about the world. It tells us nothing new.
- It is also straightforward that analytic truths are knowable a priori.
- Some mathematical truths (all triangles have three sides, all triangles are self-identical) belong here.

Mathematical truth is in general synthetic

- Conceptual analysis does not determine that 7+5 = 12, inspection of the corresponding concepts will not reveal the truth of such propositions.
- Hence the majority of mathematical truths are *synthetic*.

synthetic truth and the role of intuition

- For Kant, synthetic truths are knowable only via 'intuition' (*Anschauung*).
- Intuitions are modes of representing individual objects.
- Since mathematics presupposes reference to individuals such as numbers, mathematics needs intuition.

intuition and 'pure intuition'

- In synthetic judgments concerning empirical phenomena, perceptions are the intuitions that underlie the judgment.
- In mathematics, we do not use such empirical intuitions.
- Kant held that there is a form of intuition that yields a priori knowledge of necessary truths.

'pure intuition'

- This sort of intuition delivers the *forms of possible empirical intuitions*.
- Pure intuition is an awareness of the spatio-temporal form of ordinary sense perception.
- In mathematical reasoning, such intuitions are used for constructing the mathematical relations that underlie mathematical judgments. (Just as geometric ratios and relations are constructed in Euclidean geometry)

Kant's solution

- This explains why mathematical truth is a priori: the forms of possible empirical intuitions can be discovered by reflecting on the structure of our perceptual apparatus.

Kant's solution

- It explains why mathematics is applicable to all experiences: mathematics deals with the possible structures of *experiences*.

Kant's solution

- It explains why mathematics is necessary: it deals with the *possible* structures of experiences, not with some contingent actual structures.

Kant's solution in the light of Aristotelian Logic

- To claim that mathematics is analytic was not only hard to defend on the basis of Kant's definition of analyticity. It was also not defendable on the basis of the logic of that time.
- Kant was totally correct in claiming that logic had not made any significant progress since the time of Aristotle.

Kant's solution in the light of Aristotelian Logic

- The logic that Aristotle had established – if reconstructed in modern terms – is a monadic first-order theory with identity.
- Although some simple truths of arithmetic could be shown to be logical truths (if done properly, which it wasn't) – most of mathematics could not be established on that basis.

Kant's solution in the light of Aristotelian Logic

- Already for Euclidean geometry you'd need richer resources (you'd need second-order logic, after all).
- So, if judged by these lights, Kant was right to claim that mathematical truths cannot be established on the basis of logic (and thus on the basis of analytic considerations) alone.

Kant's solution in the light of modern logic

- Anyway, Kant was wrong, at least insofar as mathematical proofs can all be reconstructed as proofs that do not involve any "intuitive" steps (being steps not justified by the deductive system); and insofar as all mathematical truths are logical truths (if you consider second-order logic as proper logic, which I do).

Kant's solution in the light of modern logic

- That this is a plausible view, is due to the work of Frege, who helped to clean our thinking about logical relations from *psychologism* which was still part of the Kantian account.

Psychologism and the basic confusions

- One of the major problems up to Frege was the proper distinction between 'judgments', 'propositions', 'thoughts', 'sentences', and 'concepts', 'ideas', 'words', 'expressions', 'signs'
- Frege, too, did not always use them properly, but he did a good job of telling that we need to make distinctions, if we want to be precise.

Psychologism and the basic confusions

- That these notions were usually used interchangeably is an indicator for the fact that logic was not freed of psychologism. Logic was still in part a description of the psychological ways by which we arrive at certain judgments.
- However, logic is the study of logical relations between propositions that obtain independent of our contingent psychology.

Frege's Begriffsschrift

- Frege's *Begriffsschrift* already begins with making a sharp distinction between the ways we come to know a specific scientific truth and the way these truths are justified:

Frege's Begriffsschrift

"It can thus be asked, on the one hand, by what path a proposition was gradually reached, and on the other hand, in what way it is now finally to be most firmly established.
The former question possibly needs to be answered differently for different people; the latter is more definite, and it answer is connected with the inner nature of the propositions concerned."

Frege's anti-psychologism

- The same distinctions are crucial also when it comes to the objects, mathematics is dealing with. When discussing numbers in the *Grundlagen der Arithmetik*, for example, Frege ridiculed the idea that numbers could be ideas or any other mental entities, for such objects would be different for different people.

Different bases of knowledge

- The distinction between matters that can be established by logic alone and matters that need empirical evidence was of course not new. Also the idea that what can be established on the basis of logic alone is based on a firmer basis (you will find this idea in Locke, Hume, etc.). The more interesting is this:

Different bases of knowledge

"But there is no inconsistency in a proposition belonging to the first kind [whose proof can be given purely logically] and yet being such that it can never be apprehended by a human mind without the operation of the senses. Thus it is not psychological origination but the most perfect method of proof that lies at the basis of the division [between logically provable propositions and propositions that require empirical evidence]."

Frege's project

- Frege wanted to find out how much of mathematics – and of arithmetic in particular – is reducible to logic alone (and thus analytic).
- The problem of such an enterprise was that all proofs of arithmetic truths from logical principles are in danger to introduce subject matter at some point or other, if the proof is not sufficiently regimented.

The Begriffsschrift as a tool

- This problem was supposed to be solved with the *Begriffsschrift*. Frege conceived of it as an ideal language that would reveal all subject matter that would enter into a derivation.
  - "It is thus intended to serve primarily to test in the most reliable way the validity of a chain of inference and to reveal every presupposition that tends to slip in unnoticed [...]."

Begriffsschrift vs natural language

- Natural language contains too much which is totally insignificant for logical inference. It is, however, not concerned with conceptual inclusion (as Leibniz had thought of it).

Begriffsschrift vs natural language

"I believe I can make the relationship [...] clearest if I compare it to that of the microscope to the eye. The latter, due to the range of its applicability, due to the flexibility with which it is able to adapt to the most diverse circumstances, has a great superiority over the microscope...

Begriffsschrift vs natural language

"... Considered as an optical instrument, it admittedly reveals many imperfections, which usually remain unnoticed only because of its intimate connection with mental life. But as soon as scientific purposes place great demands on sharpness of resolution, the eye turns out to be inadequate. The microscope, on the other hand, is perfectly suited for just such purposes, but precisely because of this is useless for all others."

Begriffsschrift and Philosophy

"If it is a task of philosophy to break the power of words over the human mind, by uncovering illusions that through the use of language often almost unavoidably arise concerning the relation of concepts, by freeing thought from the taint of ordinary linguistic means of expression, then my *Begriffsschrift*, further developed for these purposes, can become a useful tool for philosophers."

Frege's achievement in Logic

- Indeed, Frege developed not only propositional logic, but proceeded from there up to second-order logic. (This goes beyond what we nowadays call "classical logic")
- He destroyed the dogmatic subject/predicate dichotomy that had remained in the way of thinking about logical relations since Aristotle.
- He also developed quantifiers (which before were treated like names).
- It was also – for the first time – possible to *prove* the validity of inferences that were deemed valid since Aristotle.

Frege's achievement in the philosophy of mathematics

- Frege's Logicism was destroyed by Russell's observation that the theory was inconsistent.
- The project itself is nowadays still pursued by some, but faces problems (Caesar problem, theproblem of ontology, the problem of the status of second-order logic)

Frege's achievement in philosophy

- If we only concentrate on the *Begriffsschrift*, (and leave all of the usefull new distinctions and all of the more specific claims of it aside), the *Begriffsschrift* marks the first time that a philosophical problem, by remaining a philosophical problem, was now possible to be studied by precise formal means.

Frege an the philosophy of language

- But Frege was not only a pioneer in inventing formal logic, but also in formulating questions about and highlighting problems in the semantics of natural language.

Über Sinn und Bedeutung

- Frege starts his investigation by reflecting on the identity or equality in mathematical statements. Does it express a relation between objects or between signs or names of objects?
- The problem is to explain the cognitive significance that identity statements sometimes have when they are of the form a = b.

Über Sinn und Bedeutung

- The content expressed in such statements is not merely that the objects referred to are identical, for that content would not differ from a = a, which is not cognitively significant. It also seems to be more than just telling us something about the fact that signs refer in certain ways, since these ways are arbitrary (a sign can refer to whatever we want it to refer to).

Über Sinn und Bedeutung

- Frege's idea is that the cognitive significance has to be explained with the modes of presentation that are part of the meaning of an expression. Although a and b have the same referent (Bedeutung), their modes of presentation are different, they have different *Sinne*.

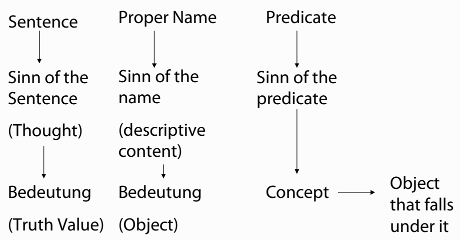
Sinn and Bedeutung of proper names

- Names refer to objects, which is their *Bedeutung*. But the descriptive content (as we would nowadays say) might be different. ‘Morning Star’ and ‘Evening Star’ refer to the same entity, but present that entity as the brightest heavenly body in the morning and the brightest heavenly body in the evening. They differ in their *Sinn*.

Sinn and Bedeutung of sentences

- Likewise, sentences have a truth value as their *Bedeutung* and a thought as their *Sinn*.
- [Predicates are complicated (they express a Sinn, refer to a concept and this relates to the objects that fall under it).]

Sinn and Bedeutung



The significance of this dichotomy

- Frege’s distinction gave rise to intensional theories of meaning. After Frege’s discovery it seemed clear that a theory of meaning could not deal with the referents only. The mode of presentation – whatever that is – must be added to get the full picture.

deviations

- This was not, however, completely captured in the following theories of intensions. Frege’s starting problem – what explains the cognitive significance of (mathematical) identity claims – was not solved at all in the theories to follow (and is hardly solved today).

Frege’s Significance

- However, the right kind of questions were now on the table and they came with the outlook for a systematic theory. It seemed clear that we could solve the problems of epistemology and ontology in mathematics if we would take more care of the ways we use ordinary and formal languages when talking about them.
- It also became clear that ordinary language and its grammatical categories were often plainly misleading, but that we could circumvent those traps by helping ourselves with formal replacements.